Eva Viehmann (TU Munich)

Newton strata in the weakly admissible locus

① The admissible locus

- period maps for p-div. gps

p prime

$\mathbb{X}$: p-div. gp. $/\overline{\mathbb{F}}_p$

$N = \mathbb{D}(\mathbb{X})_{\breve{\mathbb{Q}}_p}$ cov. Dieud. cryst. of $\mathbb{X}$

$\dim N = \mathrm{ht}\, \mathbb{X}$

$K/\breve{\mathbb{Q}}_p$, $\mathcal{O}_K$

p-adic period map

$$\pi : \left\{ (X,\rho) \;\middle|\; \begin{array}{l} X \text{ p-div gp}/\mathcal{O}_K \\ \rho: X\otimes \mathcal{O}_K/p \to \mathbb{X}\otimes \mathcal{O}_K/p \\ \text{a QI} \end{array} \right\} \longrightarrow \mathrm{Gr}_c(N)(K)$$

c ← codim $\mathbb{X}$

$$(X,\rho) \longmapsto \begin{array}{c} \mathbb{D}(X)_K \xrightarrow{\ \rho\ } N\otimes K \\ \uparrow \\ \mathrm{Lie}(X^\vee)^\vee \otimes K \end{array}$$

Qu.: Image ~~of~~ π ?

General: G reductive group over $\mathbb{Q}_p$

$b \in G(\breve{\mathbb{Q}}_p)$, μ a minuscule cochar. st. $[b] \in B(G,\mu)$

before: $\mu = (1,\dots,1,0,\dots,0)$

$\breve{\mathcal{F}}(G,\mu)$ ass. flag variety (rigid an. space)

G q'split P_μ : par. associated with μ

$$\breve{\mathcal{F}}(G,\mu) = G/P_\mu$$

$\pi : \breve{\mathcal{M}}(G, b, \mu) \longrightarrow \breve{\mathcal{F}}(G, \mu) = \breve{\mathcal{F}}$

local Shim. var — étale morph. of rigid an. sp.

$\operatorname{im} \pi =: \breve{\mathcal{F}}^a$: admissible locus

Ex.: (a) $G = GL_n$, $\mu = (1, 0, \ldots, 0)$, b basic
$\Rightarrow \breve{\mathcal{F}}^a = \breve{\mathcal{F}}(G, \mu) = \mathbb{P}^{n-1}$

(b) $G = D_{1/n}^{\times}$, $\mu = (1, 0 \ldots, 0)$, b basic

$$\breve{\mathcal{F}}^a = \Omega = \mathbb{P}^{n-1} \setminus \bigcup_{\substack{H:\, \mathbb{Q}_p\text{-rat} \\ \text{hyperplane}}} H \subseteq \breve{\mathcal{F}}(G, \mu) = \mathbb{P}^{n-1}$$

② The weakly adm. locus

$\breve{\mathcal{F}}^a \underset{\text{open}}{\subset} \breve{\mathcal{F}}^{wa} \underset{\text{open}}{\subset} \breve{\mathcal{F}}$ $\qquad$ $\breve{\mathcal{F}}^{wa}$: weakly adm. locus

$\rightarrow$ remove from $\breve{\mathcal{F}}$ a profinite union of translates of Schubert vas.

Ex.: (a), (b) above: $\breve{\mathcal{F}}^a = \breve{\mathcal{F}}^{wa}$

Colmez-Fontaine: $K / \breve{\mathbb{Q}}_p$ finite $\Rightarrow \breve{\mathcal{F}}^a(K) = \breve{\mathcal{F}}^{wa}(K)$

Hartl: not equal in gen.

Chen-Fargues-Shen: $\breve{\mathcal{F}}^a = \breve{\mathcal{F}}^{wa} \Leftrightarrow$
(G, μ) is fully HN-decomposable

③ Newton strata

$C / \overline{\mathbb{Q}_p}$ alg. closed, complete, $C^\flat$: ~~is~~ its tilt

$\Rightarrow$ Fargues-Fontaine curve $X \ni \infty$

with $k(\infty) = C$, $\hat{\mathcal{O}}_{X,\infty} = B^+_{dR}(C)$

Fargues:

$$\mathcal{B}(G) = \{\sigma\text{- conj. cl. of } G(\breve{\mathbb{Q}}_p)\} \longleftrightarrow \{G\text{-bundles on } X\}_{/\cong}$$

$$[b] \longmapsto \mathcal{E}_b$$

$[b] \in B(G) \xleftrightarrow{\text{Kottwitz}}$
- $\nu_b \in X_*(A)_{\mathbb{Q},dom}$ "Newton pt"
- $\kappa_G(b) \in \pi_1(G)_\Gamma$ "Kottwitz pt."

$x \in \breve{\mathcal{F}}(G,\mu)(C)$, $[b] \in B(G)$ basic

modif. $\mathcal{E}_{b,x}$ of $\mathcal{E}_b$ at ∞ Beauville - Laszlo

glue: $\mathcal{E}_b|_{X \setminus \{\infty\}}$, triv. G-bundle over Spec $B^+_{dR}(C)$

gluing datum $\triangleq x$

$$\breve{\mathcal{F}}(G,\mu) = \coprod_{[b'] \in B(G)} \mathcal{F}(G,\mu,b)^{[b']}$$

locus where $\mathcal{E}_{b,x} \simeq \mathcal{E}_{b'}$

"Newton strata"

- $[b] \in B(G,\mu) \Longrightarrow \breve{\mathcal{F}}^a = \breve{\mathcal{F}}(G,\mu,b)^{[1]}$ open Newton stratum

- Caraiani - Scholze, Rapoport
  $\breve{\mathcal{F}}(G,\mu,b)^{[b']} \neq \emptyset \Leftrightarrow [b'] \in B(G,\mu,b)$ i.e.
  - $\kappa_G(b') = \kappa_G(b) - \mu^\#$
  - $\nu_{b'} \leq \nu_b(\mu^{-1})^\diamond_{dom}$

4 Intersections

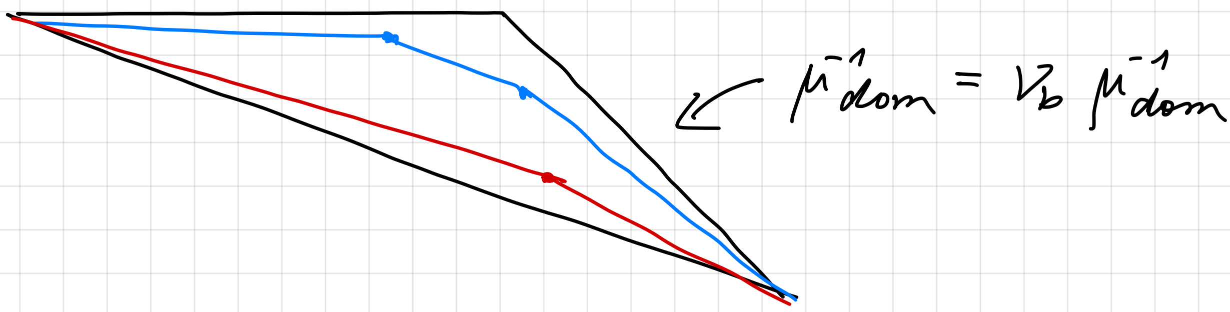
Conjecture (Chen)

$$\breve{\mathcal{F}}^{wa} \cap \mathcal{F}(G,b,\mu)^{[b']} \neq \emptyset$$

$\Leftrightarrow [b'] \in B(G,b,\mu)$ is Hodge-Newton indec

$G = GL_n$

$[b] = 1, \quad \mu = (1, \dots, 1, 0, \dots 0)$



Known cases:

"$\Rightarrow$" [Chen-Fargues-Shen]

"$\Leftarrow$" [CFS, Chen] for minimal non-basic elements of $B(G, \mu, b)$

[Chen] some other "small" $[b']$, $G = GL_n$

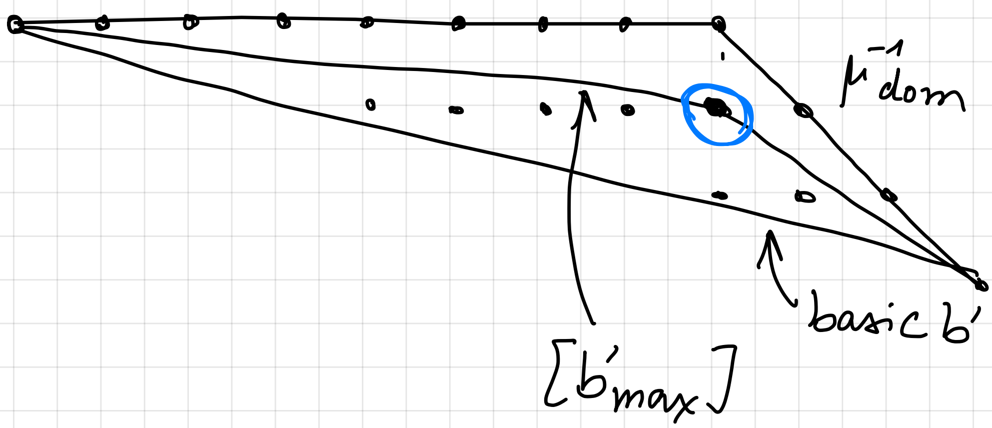
Thm (V) The conjecture holds for $G = GL_n$, $[b] = [1]$

Ideas for the proof:

- For GL_n we have [Hansen; Birkbeck et. al.]

$$\overline{\mathcal{F}(G, \mu, b)^{[b']}} = \bigcup_{[b''] \geq [b']} \mathcal{F}(G, \mu, b)^{[b'']}$$

$\Rightarrow$ enough to consider the conj. for the unique maximal HN-indec. $[b'] \in B(G, \mu, b)$



▷ Assume $x \in \check{\mathcal{F}}(G,\mu,b)^{[b'_{max}]}$ not w.a.

$\Rightarrow$ reduction $(\mathcal{E}_{b,x})_P$ coming from red. of b to P with "positive slope pol"

P ↑ parab.

▷ [Chen] $(\mathcal{E}_{b,x})_P \times_P M \longleftrightarrow$ σ-conj. class for M

ν_M : Newton pt.

$\mu_{dom}^{-1} \geq (\nu_M)_{G\text{-dom}} \geq \nu_{b'_{max}}$ $\Longrightarrow$ 2 cases : $\underline{\underline{(\nu_M)_{G\text{-dom}} = \nu_{b'_{max}}}}$ or HN-dec.

▷ HN-decomposition .... $\Rightarrow$ cannot occur

$$\Longrightarrow \nu_M = \nu_{b'_{max}}$$

$$\xRightarrow{CFS} x \in \underbrace{PwP_\mu/P_\mu}_{\dim:\ n-1} \subset G/P_\mu \qquad w = s_i$$

$$\dim \check{\mathcal{F}}(G,\mu,b)^{[b'_{max}]} = \langle 2\rho, \mu - \nu_{b'_{max}} \rangle = n$$