Restricting Representations via Restricting G-data and Kim-Yu Types

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Notation and Goal

- $\mathbb{G}$ reductive group defined over p-adic field F, $G = \mathbb{G}(F)$
- $\mathbb{H}$ reductive F-subgroup of $\mathbb{G}$ that contains $[\mathbb{G}, \mathbb{G}]$

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Goal: Take an irreducible representation π of G and study $\pi|_H$.

** with a few hypotheses

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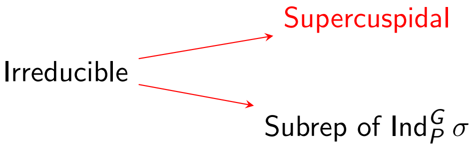

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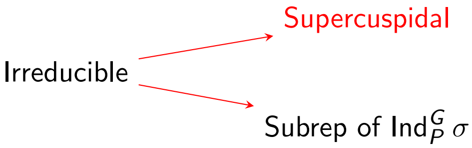

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Fintzen: Yu's construction is exhaustive!

A First Observation...

$$\pi_G(\Psi)|_H \quad = \quad \pi_1 \quad \oplus \quad \pi_2 \quad \oplus \quad \cdots \quad \oplus \quad \pi_m$$

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\end{array}$$

What is the relationship between the G-datum Ψ and the H-data Ψ_i?

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- $\vec{\phi} = (\phi^0, \phi^1, \ldots, \phi^d)$ is a sequence of generic quasicharacters of depth r_i for $0 \leq i \leq d-1$. If $r_{d-1} < r_d$, we assume ϕ^d is of depth r_d, otherwise $\phi^d = 1$.

Restricting a G-datum

$$(\mathbb{G}^0, \quad \mathbb{G}^1, \quad \ldots, \quad \mathbb{G}^d), \qquad \rho, \qquad (\phi^0, \quad \ldots, \quad \phi^{d-1}, \quad \phi^d)$$

Restricting a G-datum

$$
\begin{array}{ccccccc}
(\mathbb{G}^0, & \mathbb{G}^1, & \dots, & \mathbb{G}^d), & \rho, & (\phi^0, \quad \dots, \quad \phi^{d-1}, \quad \phi^d) \\
\downarrow {\scriptstyle \cap \mathbb{H}} & \downarrow {\scriptstyle \cap \mathbb{H}} & & \downarrow {\scriptstyle \cap \mathbb{H}} & & \\
(\mathbb{H}^0, & \mathbb{H}^1, & \dots, & \mathbb{H}^d), & &
\end{array}
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\downarrow{\scriptstyle \cap\mathbb{H}} & \downarrow{\scriptstyle \cap\mathbb{H}} & & \downarrow{\scriptstyle \cap\mathbb{H}} & \vdots & & \\
(\mathbb{H}^0, & \mathbb{H}^1, & \ldots, & \mathbb{H}^d), & \rho_\ell, & &
\end{array}
$$

an irreducible component of $\rho|_{H^0_{[y]}}$

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(\mathbb{G}^0, & \mathbb{G}^1, & \ldots, & \mathbb{G}^d), & \rho, & (\phi^0, & \ldots, & \phi^{d-1}, & \phi^d) \\
\downarrow \cap\mathbb{H} & \downarrow \cap\mathbb{H} & & \downarrow \cap\mathbb{H} & \vdots & \mathrm{Res}_{H^0}^{G^0} \downarrow & & \mathrm{Res}_{H^{d-1}}^{G^{d-1}} \downarrow & \downarrow \mathrm{Res}_{H^d}^{G^d} \\
(\mathbb{H}^0, & \mathbb{H}^1, & \ldots, & \mathbb{H}^d), & \rho_\ell, & (\phi_H^0, & \ldots, & \phi_H^{d-1}, & \phi_H^d)
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$$

$\tilde{r} \leq r_{d-1}$

Theorem (B)

Let $\tilde{r}$ denote the depth of ϕ_H^d.

1) *If $\tilde{r} > r_{d-1}$, set $\vec{\phi}_H = (\phi_H^0, \ldots, \phi_H^{d-1}, \phi_H^d)$ and $\vec{\tilde{r}} = (r_0, \ldots, r_{d-1}, \tilde{r})$.*
2) *If $\tilde{r} \leq r_{d-1}$, set $\vec{\phi}_H = (\phi_H^0, \cdots, \phi_H^{d-1}\phi_H^d, 1)$ and $\vec{\tilde{r}} = (r_0, \ldots, r_{d-1})$.*

Then, $\Psi_\ell = (\vec{\mathbb{H}}, y, \vec{\tilde{r}}, \rho_\ell, \vec{\phi}_H)$ is an H-datum.

What we expect...

$$\pi_G(\Psi)$$

$$\Big|$$

$$\left(\vec{\mathbb{G}}, y, \vec{r}, \rho, \vec{\phi}\right)$$

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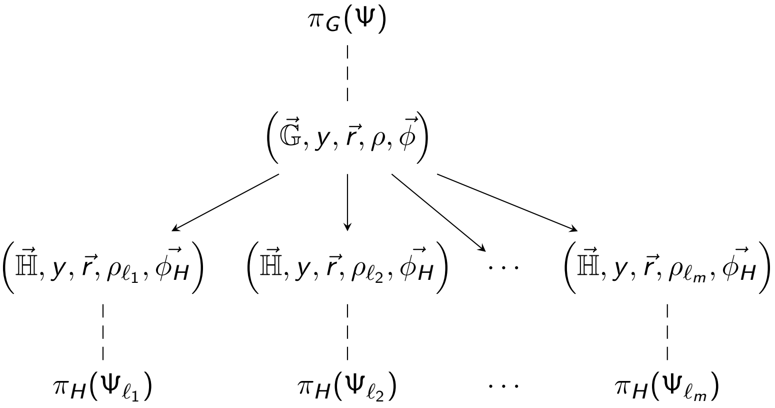
$$\left(\vec{\mathbb{H}}, y, \vec{r}, \rho_{\ell_1}, \vec{\phi_H}\right) \quad \left(\vec{\mathbb{H}}, y, \vec{r}, \rho_{\ell_2}, \vec{\phi_H}\right) \quad \cdots \quad \left(\vec{\mathbb{H}}, y, \vec{r}, \rho_{\ell_m}, \vec{\phi_H}\right)$$

What we expect...



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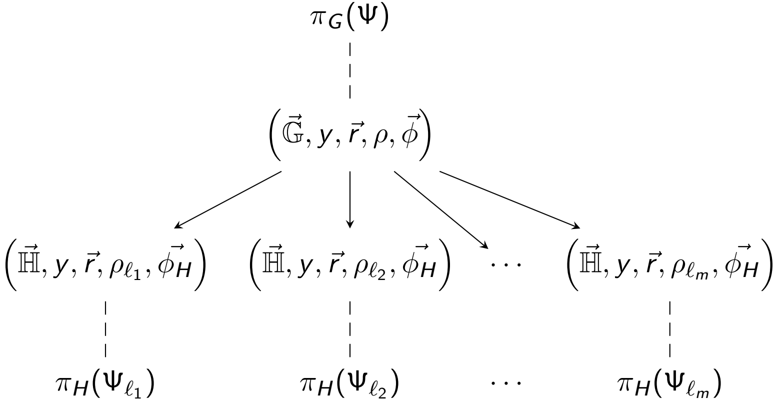

$$\Rightarrow \pi_H(\Psi_{\ell_i}) \subset \pi_G(\Psi)|_H, 1 \leq i \leq m.$$

J.K. Yu Construction

$$\left(\phi^0, K^0\right) \quad \cdots \quad \left(\phi^{d-2}, K^{d-2}\right) \quad \left(\phi^{d-1}, K^{d-1}\right) \quad \left(\phi^d, K^d\right)$$

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\downarrow \text{extend} & & \downarrow \text{extend} & \downarrow \text{extend} & \downarrow = \\
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\end{array}
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$$\kappa_G(\Psi) = \kappa^{-1} \otimes \kappa^0 \otimes \kappa^1 \otimes \kappa^2 \cdots \otimes \kappa^{d-2} \otimes \kappa^{d-1} \otimes \kappa^d$$

Restriction of $\pi_G(\Psi)$

$$\begin{array}{c} (\rho, K^0) \\ \text{inflate} \downarrow \\ (\kappa^{-1}, K^d) \end{array}$$

$$\begin{array}{c} (\phi^i, K^i) \\ \text{extend} \downarrow \\ \left(\phi^{i\prime}, K^{i+1}\right) \\ \text{inflate} \downarrow \\ (\kappa^i, K^d) \end{array}$$

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$$
\begin{array}{ccc}
(\rho, K^0) & \xrightarrow{\text{restrict}} & \left(\bigoplus_{\ell \in L} \rho_\ell, K_H^0\right) \\
\text{inflate} \downarrow & & \downarrow \text{inflate} \\
(\kappa^{-1}, K^d) & & \left(\bigoplus_{\ell \in L} \kappa_\ell^{-1}, K_H^d\right)
\end{array}
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$$\begin{array}{ccc} (\phi^i, K^i) & \xrightarrow{\text{restrict}} & (\phi^i_H, K^i_H) \\ \text{extend}\downarrow & \circlearrowleft & \downarrow\text{extend} \\ \left(\phi^{i\prime}, K^{i+1}\right) & \xrightarrow[\text{restrict}]{} & \left({\phi^i_H}', K_H^{i+1}\right) \\ \text{inflate}\downarrow & \circlearrowleft & \downarrow\text{inflate} \\ (\kappa^i, K^d) & \xrightarrow[\text{restrict}]{} & (\kappa^i_H, K^d_H) \end{array}$$

Theorem (B)

$$\kappa_G(\Psi)|_{K_H^d} = \bigoplus_{\ell\in L}\kappa_H(\Psi_\ell)$$

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Theorem (B)

$\kappa_G(\Psi)|_{K^d_H} = \bigoplus_{\ell \in L} \kappa_H(\Psi_\ell)$ and $\pi_G(\Psi)|_H \simeq \bigoplus_{t \in C} \left(\bigoplus_{\ell \in L} \pi_H({}^t\Psi_\ell) \right)$, where C is a set of representatives of $H \backslash G / K^d$.

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No analog of Mautner's Theorem, let's use types!

Defining Types

Bernstein Decomposition:

$$\mathcal{R}(G) = \prod_{\mathfrak{s} \in \mathfrak{B}} \mathcal{R}^{\mathfrak{s}}(G).$$

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We say that an irreducible smooth representation π of G contains a type if there exists an $\mathfrak{s}$-type (J,λ) such that $\pi|_J$ contains λ.

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Fintzen: Every irreducible representation π of G contains a Kim-Yu type!

Definition of Type-datum

A sequence $\Sigma = ((\vec{\mathbb{G}}, \mathbb{M}^0), (y, \{I\}), \vec{r}, (M_y^0, \rho), \vec{\phi})$ is a G type-datum for G iff:

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- y in $\mathcal{B}(\mathbb{G}, F) \cap \mathcal{A}(\mathbb{G}, \mathbb{T}, E)$ and $\{I\}$ is a generic embedding;
- ρ is an irreducible representation of M^0_y such that $\rho|_{M^0_{y,0}}$ contains a cuspidal;

- $\vec{r} = (r_0, r_1, \ldots, r_d)$ is a sequence of real numbers satisfying $0 < r_0 < r_1 < \cdots < r_{d-1} \leq r_d$ if $d > 0$, $0 \leq r_0$ if $d = 0$;

Definition of Type-datum

A sequence $\Sigma = ((\vec{\mathbb{G}}, \mathbb{M}^0), (y, \{I\}), \vec{r}, (M_y^0, \rho), \vec{\phi})$ is a type-datum for G iff:

- $\vec{\mathbb{G}} = (\mathbb{G}^0, \mathbb{G}^1, \ldots, \mathbb{G}^d)$ and $\mathbb{M}^0$ is a Levi subgroup of $\mathbb{G}^0$;
- y in $\mathcal{B}(\mathbb{G}, F) \cap \mathcal{A}(\mathbb{G}, \mathbb{T}, E)$ and $\{I\}$ is a generic embedding;
- ρ is an irreducible representation of M_y^0 such that $\rho|_{M_{y,0}^0}$ contains a cuspidal;
- $\vec{r} = (r_0, r_1, \ldots, r_d)$ is a sequence of real numbers satisfying $0 < r_0 < r_1 < \cdots < r_{d-1} \leq r_d$ if $d > 0$, $0 \leq r_0$ if $d = 0$;
- $\vec{\phi} = (\phi^0, \phi^1, \ldots, \phi^d)$ is a sequence of generic quasicharacters of depth r_i for $0 \leq i \leq d-1$. If $r_{d-1} < r_d$, we assume ϕ^d is of depth r_d, otherwise $\phi^d = 1$.

Restriction of Type-datum

$$(\vec{\mathbb{G}}, \quad \mathbb{M}^0), \qquad \rho, \qquad (\phi^0, \quad \ldots, \quad \phi^{d-1}, \quad \phi^d)$$

Restriction of Type-datum

$$\begin{array}{ccccccc} (\vec{\mathbb{G}}, & \mathbb{M}^0), & \rho, & (\phi^0, & \dots, & \phi^{d-1}, & \phi^d) \\ \big\downarrow {\scriptstyle \cap\mathbb{H}} & \big\downarrow {\scriptstyle \cap\mathbb{H}} & & & & & \\ (\vec{\mathbb{H}}, & \mathbb{M}^0_{\mathbb{H}}), & & & & & \end{array}$$

Restriction of Type-datum

$$
\begin{array}{ccccccc}
(\vec{\mathbb{G}}, & \mathbb{M}^0), & \rho, & (\phi^0, & \ldots, & \phi^{d-1}, & \phi^d) \\
\downarrow \cap\mathbb{H} & \downarrow \cap\mathbb{H} & \vdots & & & & \\
(\vec{\mathbb{H}}, & \mathbb{M}^0_{\mathbb{H}}), & \rho_\ell, & & & &
\end{array}
$$

an irreducible component of $\rho|_{(M^0_H)_y}$

Restriction of Type-datum

$$
\begin{array}{cccccccc}
(\vec{\mathbb{G}}, & \mathbb{M}^0), & \rho, & (\phi^0, & \ldots, & \phi^{d-1}, & \phi^d) \\
\big\downarrow \cap\mathbb{H} & \big\downarrow \cap\mathbb{H} & \vdots & \color{red}{\mathrm{Res}_{H^0}^{G^0}} \big\downarrow & & \color{red}{\mathrm{Res}_{H^{d-1}}^{G^{d-1}}} \big\downarrow & \big\downarrow \color{red}{\mathrm{Res}_{H^d}^{G^d}} \\
(\vec{\mathbb{H}}, & \mathbb{M}^0_{\mathbb{H}}), & \rho_\ell, & (\phi^0_H, & \ldots, & \phi^{d-1}_H, & \phi^d_H)
\end{array}
$$

Restriction of Type-datum

$$
\begin{array}{ccccccc}
(\vec{\mathbb{G}}, & \mathbb{M}^0), & \rho, & (\phi^0, & \ldots, & \phi^{d-1}, & \phi^d) \\
\downarrow \cap\mathbb{H} & \downarrow \cap\mathbb{H} & \vdots & \mathrm{Res}^{G^0}_{H^0}\downarrow & & \mathrm{Res}^{G^{d-1}}_{H^{d-1}}\downarrow & \downarrow \mathrm{Res}^{G^d}_{H^d} \\
(\vec{\mathbb{H}}, & \mathbb{M}^0_{\mathbb{H}}), & \rho_\ell, & (\phi^0_H, & \ldots, & \phi^{d-1}_H, & \phi^d_H) \\
 & & & & & \curvearrowleft & \\
 & & & & & \tilde{r} \leq r_{d-1} &
\end{array}
$$

Theorem (B)

Let $\tilde{r}$ denote the depth of ϕ^d_H.

1) *If $\tilde{r} > r_{d-1}$, set $\vec{\phi}_H = (\phi^0_H, \ldots, \phi^{d-1}_H, \phi^d_H)$ and $\vec{\tilde{r}} = (r_0, \ldots, r_{d-1}, \tilde{r})$.*
2) *If $\tilde{r} \leq r_{d-1}$, set $\vec{\phi}_H = (\phi^0_H, \cdots, \phi^{d-1}_H \phi^d_H, 1)$ and $\vec{\tilde{r}} = (r_0, \ldots, r_{d-1})$.*

Then, $\Sigma_\ell = ((\vec{\mathbb{H}}, \mathbb{M}^0_{\mathbb{H}}), (y, \{I\}), \vec{\tilde{r}}, ((M^0_H)_y, \rho_\ell), \vec{\phi}_H)$ is a type-datum for H.

Restriction of Type

$$(J, \lambda_G(\Sigma))$$

$$\big((\vec{\mathbb{G}}, \mathbb{M}^0), (y, \{I\}), \vec{r}, (M_y^0, \rho), \vec{\phi}\big)$$

Restriction of Type

$$(J, \lambda_G(\Sigma))$$

$$\Big((\vec{\mathbb{G}}, \mathbb{M}^0), (y, \{\iota\}), \vec{r}, (M_y^0, \rho), \vec{\phi}\Big)$$

$$\downarrow$$

$$\left\{ \Big((\vec{\mathbb{H}}, \mathbb{M}^0_{\mathbb{H}}), (y, \{\iota\}), \vec{r}, ((M_H^0)_y, \rho_\ell), \vec{\phi_H}\Big), \ell \in L \right\}$$

Restriction of Type

$$(J, \lambda_G(\Sigma))$$

$$\Big((\vec{\mathbb{G}}, \mathbb{M}^0), (y, \{I\}), \vec{r}, (M_y^0, \rho), \vec{\phi}\Big)$$

$$\downarrow$$

$$\Big\{\Big((\vec{\mathbb{H}}, \mathbb{M}^0_{\mathbb{H}}), (y, \{I\}), \vec{r}, ((M_H^0)_y, \rho_\ell), \vec{\phi_H}\Big), \ell \in L\Big\}$$

$$\{(J_H, \lambda_H(\Sigma_\ell)), \ell \in L\}$$

Restriction of Type

$$(J, \lambda_G(\Sigma))$$

$$\Big((\vec{\mathbb{G}}, \mathbb{M}^0), (y, \{I\}), \vec{r}, (M_y^0, \rho), \vec{\phi}\Big)$$

$$\downarrow$$

$$\Big\{\Big((\vec{\mathbb{H}}, \mathbb{M}^0_{\mathbb{H}}), (y, \{I\}), \vec{r}, ((M_H^0)_y, \rho_\ell), \vec{\phi_H}\Big), \ell \in L\Big\}$$

$$\{(J_H, \lambda_H(\Sigma_\ell)), \ell \in L\}$$

Theorem (B)

$$\lambda_G(\Sigma)|_{J_H} = \bigoplus_{\ell \in L} \lambda_H(\Sigma_\ell)$$

Restriction of the Representation

Let π be an irrep of G.

- $(J, \lambda_G(\Sigma)) \subset \pi$

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Let π be an irrep of G.

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Restriction of the Representation

Let π be an irrep of G.

- $(J, \lambda_G(\Sigma)) \subset \pi \Rightarrow \{(J_H, \Sigma_\ell), \ell \in L\} \subset \pi|_H$
- theory of types does not give correspondence with reps, so open problems!

Thank you

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